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ON TOTAL EDGE IRREGULAR STRENGTH OF TRIANGLE RELATED GRAPHS

FATMA SALAMA[1], MUHAMMAD KAMRAN SIDDIQUI[2,*]
[1] Mathematics Department, Faculty of Science, Tanta University, Tanta, Egypt
[2]Department of Mathematics, COMSATS University Islamabad, Lahore Campus, 54000, Pakistan

Abstract. An edge irregular total h -labeling of a simple, undirected and connected graph $G(V,E)$ is a labeling defined by $H : V \cup E \to \{1,2,3,\ldots,h\}$ so that for any two distinct edges pq, rs their weights are different; i.e. $W_H(pq) \neq W_H(rs)$ where $W_H(pq) = H(p) + H(q) + H(pq)$. A total edge irregular strength [TEIS] of a graph G, denoted by $tes(G)$, is a labelling with the minimum h. In this paper, we have calculated the exact value of TEIS of triangle related graphs namely half gear graph, double fan graph and triangular snake graph.

Keywords: Total edge irregularity strength, Half gear graph, Double fan graph, Triangular snake graph.

1. Introduction

A function which assign some set of elements of a graph G with a set of positive or non-negative integers is called labeling. Authors in [1] have defined an edge irregular total h-labeling for a simple, connected and undirected graph $G(V,E)$ as a labelling $H : V \cup E \to \{1,2,3,\ldots,h\}$ such that a graph G have distinct weights, for any two distinct edges l and l^* i.e. $w_H(l) \neq w_H(l^*)$. If a graph G admits an edge irregular total $h-$labeling and h is the minimum, then $tes(G)$ is a of a graph TEIS G. Moreover, for any graph G, $tes(G)$ is given by [1]:

$$tes(G) \geq \max\{\lceil \frac{|E(G)|+2}{3} \rceil, \lceil \frac{\Delta(G)+1}{2} \rceil\}. \tag{1}$$

Conjecture 1: For any graph G , except K_5 ,with maximum degree $\Delta(G)$ the following relation is satisfied:

$$tes(G) = \max\{\lceil \frac{|E(G)|+2}{3} \rceil, \lceil \frac{\Delta(G)+1}{2} \rceil\}.$$

The above conjecture has been made out for polar grid graph [2], for some Cartesian product graphs in [3], for a categorical product of two paths [4], for categorical product of two cycles in [5], for subdivision of star [6], for dense graphs [7], for generalized prism in [8], for large graphs in [9], for generalized web graphs and related graphs [10], for zigzag graphs [11], for hexagonal grid graphs in [12], for fan graph, wheel graph, triangular book graph, friendship graph [13], for centralized uniform theta graphs. For more details, see [14-27].

*Correspondence: kamransiddiqui75@gmail.com
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In this paper, the exact value of TEIS for half gear graph, double fan graph and triangular snake graph are determined.

2. Main results

2.1. Total edge irregularity strength of half gear graph.

The half gear graph HG_n, is the graph obtained from the fan graph $F_n$by inserting a vertex between any two adjacent vertices in its path P_n [28].

Theorem 1. Let $m \geq 2$, HG_m be a half gear graph with $2m$ vertices and $3m-2$ edges. Then

$$tes(HG_m) = m.$$

Proof: Since $|V(HG_m)| = 2m$, $|E(HG_m)| = 3m-2$, then by replacement in (1) we have

$$tes(HG_m) \geq \lceil \frac{3m-2+2}{3} \rceil = m.$$

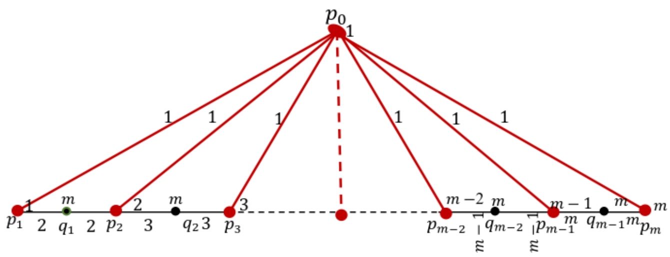

Figure (1) Half gear graph HG_m

Now, it's necessary to prove that there exists an edge irregular total h- labeling for HG_m , in Figure (1), with $h = m$. Let $h = m$ and a total h- labeling $H : V \cup E \to \{1,2,3,\ldots,h\}$ defined as:

$$\begin{aligned}
H(p_0) &= 1, \\
H(p_i) &= i, \quad \text{for } 1 \leq i \leq m, \\
H(q_i) &= h, \quad \text{for } 1 \leq i \leq m-1, \\
H(p_i q_i) &= i+1, \quad \text{for } 1 \leq i \leq m-1, \\
H(q_i p_{i+1}) &= i+1, \quad \text{for } 1 \leq i \leq m-1, \\
H(p_0 p_i) &= 1, \quad \text{for } 1 \leq i \leq m.
\end{aligned}$$

It is obvious that the greatest label is $h = n$. Also, the weights of the edges of a gear graph G_m are given by:

$$\begin{aligned} W_H(p_0p_i) &= i+2, \quad \text{for } 1 \le i \le m, \\ W_H(p_iq_i) &= 2i+h+1, \quad \text{for } 1 \le i \le m-1, \\ W_H(q_ip_{i+1}) &= 2i+h+2, \quad \text{for } 1 \le i \le m-1. \end{aligned}$$

Upon checking, it was found that the weights of any two different edges in HG_m are different. Hence, H is an edge irregular total h- labeling; $h = m$; for the half gear graph HG_m, i.e.

$$tes(HG_m) = m.$$

Example 2.1. *For the half gear graphs HG_4, HG_6 and HG_9 the TEISs of them are given by $tes(HG_4) = 4$, $tes(HG_6) = 6$, $tes(HG_9) = 9$. See Figures (2), (3) and (4).*

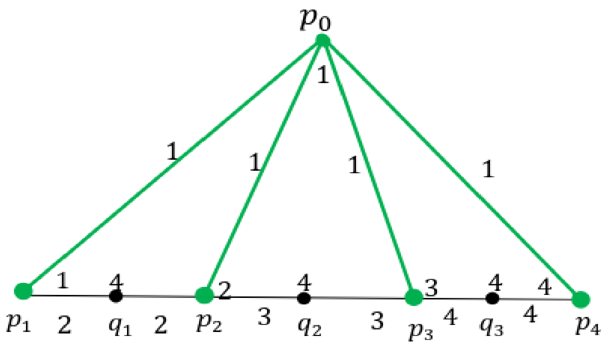

Figure (2) Half gear graph HG_4

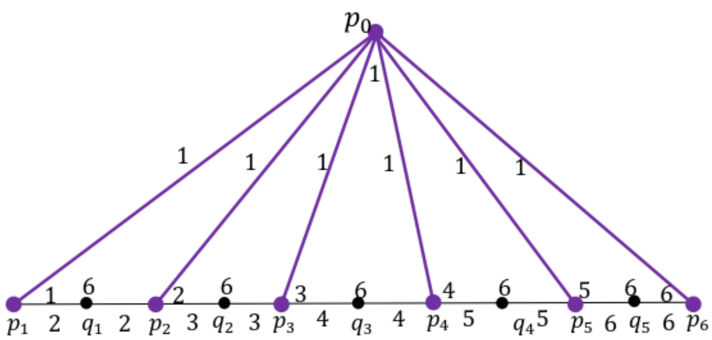

Figure (3) Half gear graph HG_6

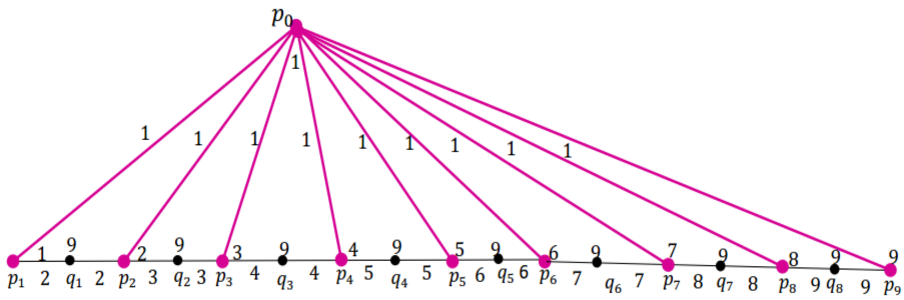

Figure (4) Half gear graph HG_9

2.2. Total edge irregularity strength of a double fan graph $F_{2,m} = \bar{K}_2 + P_m$.

Theorem 2.2. *Let $F_{2,m} = \bar{K}_2 + P_m$ be a double fan graph with order $m+2$ and size $3m-1, m \geq 2$. Then*

$$tes(F_{2,m}) = m+1.$$

Proof. Since $|V| = m+2$, $|E| = 3m-1$, then form (1),

$$\begin{aligned} tes(F_{2,m}) &\geq \lceil \frac{3m+1}{3} \rceil, \\ &= \lceil \frac{3m+3}{3} - \frac{2}{3} \rceil, \\ &= \lceil m+1-\frac{2}{3} \rceil, \\ &= \lceil m+\frac{1}{3} \rceil = m+1. \end{aligned}$$

Thus, $tes(F_{2,m}) \geq m+1$.

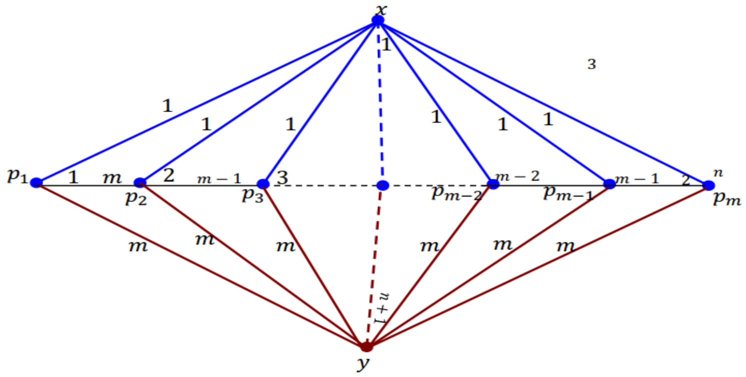

Figure (5) Double fan graph

Since we need to show that there exists an edge irregular total t-labeling with $t = m+1$ for a fan graph $F_{2,m}$ shown in Figure (5). Let $t = m+1$ and $T : V \cup E \to \{1,2,3,\ldots,t\}$ is a total t - labeling and can be defined as:

$$\begin{aligned} T(x) &= 1, \\ T(y) &= t, \\ T(p_i) &= i, \text{ for } 1 \le i \le m, \\ T(xp_i) &= 1, \text{ for } 1 \le i \le m, \\ T(yp_i) &= t-1, \text{ for } 1 \le i \le m, \\ T(p_ip_{i+1}) &= t-i, \text{ for } 1 \le i \le m-1. \end{aligned}$$

Obviously, $t = m+1$ is the greatest label. On the other hand, weights of edges in $F_{2,m}$ are given by:

$$\begin{aligned} W_T(xp_i) &= 2+i, \text{ for } 1 \le i \le m, \\ W_T(yp_i) &= 2t+i-1, \text{ for } 1 \le i \le m, \\ W_T(p_ip_{i+1}) &= i+t+1, \text{ for } 1 \le i \le m-1. \end{aligned}$$

From the above equations it's clear that the weights of any two distinct edges are distinct. Hence T is an edge irregular total t - labeling, $t = m+1$. This mean that

$$tes(F_{2,m}) = m+1.$$

□

Example 2. TEIS of $F_{2,5}$, $F_{2,6}$ and $F_{2,8}$ are $tes(F_{2,5}) = 6$, $tes(F_{2,6}) = 7$, $tes(F_{2,8}) = 9$, see Figures (6), (7) and (8).

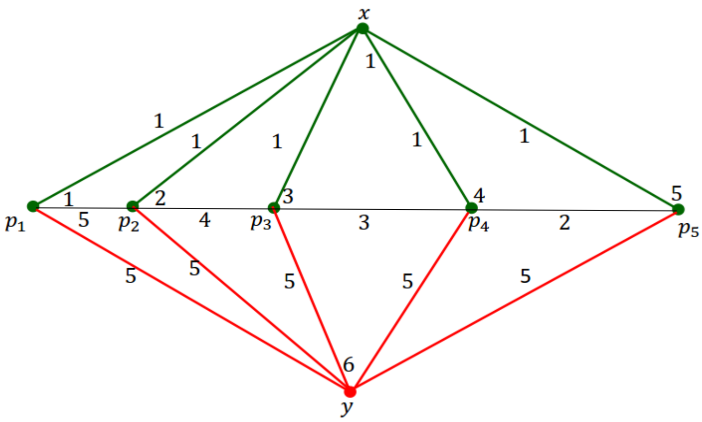

Figure (6) A double fan graph $F_{2,5}$

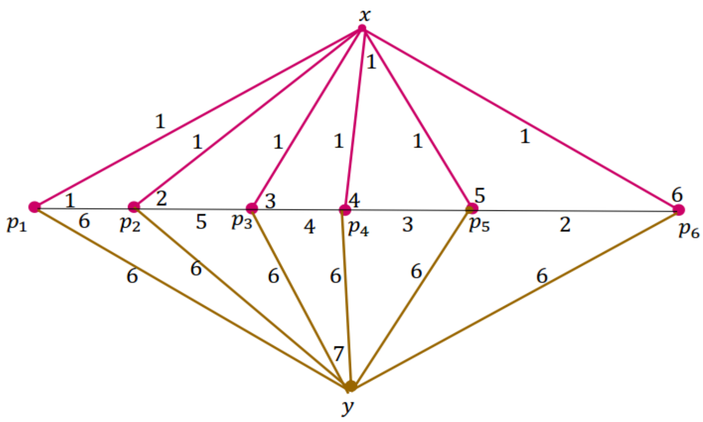

Figure (7) A double fan graph $F_{2,6}$

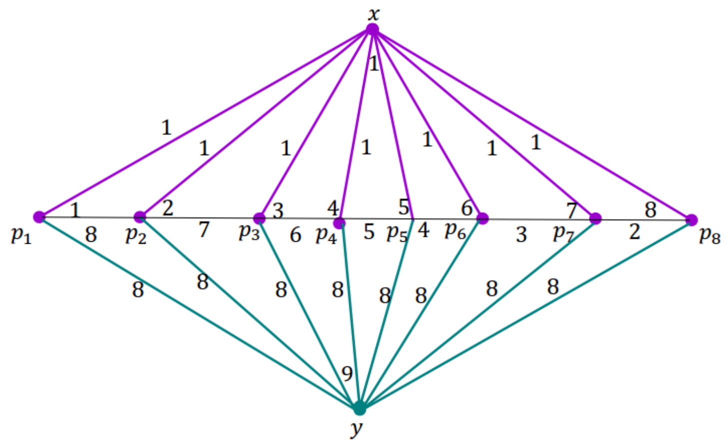

Figure (8) A double fan graph $F_{2,8}$

2.3. **Total edge irregularity strength of triangular snake graph T_m.**

The triangular snake T_n is obtained from the path P_n by replacing each edge of the path by a Triangle C_3 [29].

Theorem 2.3. *For any triangular snake graphT_m with an order of $2m-1$ and a size of $3(m-1)$, $m \geq 2$, then:*

$$tes(T_m) = m.$$

Proof. Since $|V(T_m)| = 2m-1$, $|E(T_m)| = 3(m-1)$, then from (1),

$$\begin{aligned} tes(T_m) &\geq \lceil \frac{3m-3+2}{3} \rceil \\ &= \lceil \frac{3m-1}{3} \rceil \\ &= \lceil m - \frac{1}{3} \rceil \\ &= m. \end{aligned}$$

Thus, $tes(T_m) \geq m$.

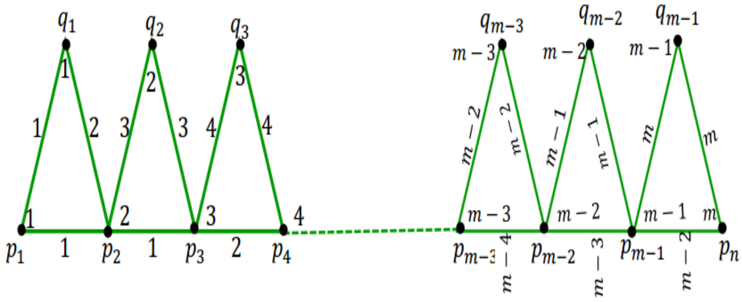

Figure.(9) triangular snake graph T_m

Now, we show that there exist an edge irregular total λ-labeling with $\lambda = m$. Let $\lambda = m$ and $Y : V \cup E \to \{1, 2, 3, \ldots, \lambda\}$ is a total λ-labeling defined by:

$$\begin{aligned}
Y(p_i) &= i, \text{ for } 1 \le i \le m, \\
Y(q_i) &= i, \text{ for } 1 \le i \le m-1, \\
Y(p_1p_2) &= 1, \\
Y(p_1q_1) &= 1, \\
Y(p_ip_{i+1}) &= i-1, \text{ for } 2 \le i \le m-1, \\
Y(q_ip_{i+1}) &= i+1, \text{ for } 1 \le i \le m-1, \\
Y(q_ip_i) &= i+1, \text{ for } 2 \le i \le m-1.
\end{aligned}$$

Obviously, the greatest value of the label is $\lambda = m$. The weights of the edges of T_m are:

$$\begin{aligned}
W_Y(p_1p_2) &= 4, \\
W_Y(p_1q_1) &= 3, \\
W_Y(p_ip_{i+1}) &= 3i, \text{ for } 2 \le i \le m-1, \\
W_Y(q_ip_{i+1}) &= 3i+2, \text{ for } 1 \le i \le m-1, \\
W_Y(q_ip_i) &= 3i+1, \text{ for } 2 \le i \le m-1.
\end{aligned}$$

Checking the weights of edges clarifies that they are different. Hence, Y is total irregularity strength for T_m ,see figure (9), i.e.

$$tes(T_m) = m.$$

□

Example 2.4. *TEIS of the triangular snake graphs T_6, T_8, T_9 are given $tes(T_6) = 6$, $tes(T_8) = 8$, $tes(T_9) = 9$. See Figures (10), (11) and (12).*

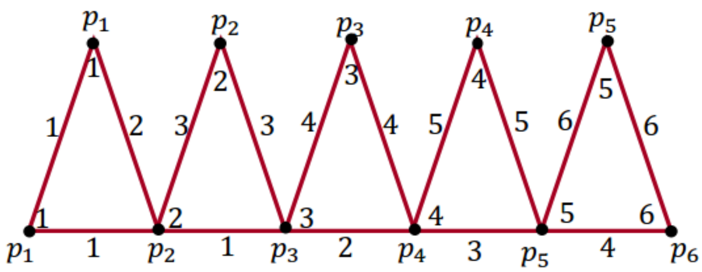

Figure (10) The triangular snake graph T_6

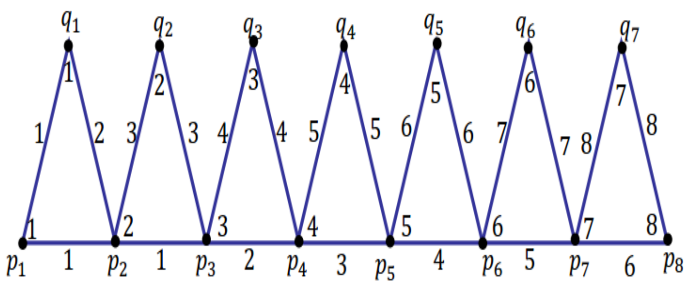

Figure (11) The triangular snake graph T_8

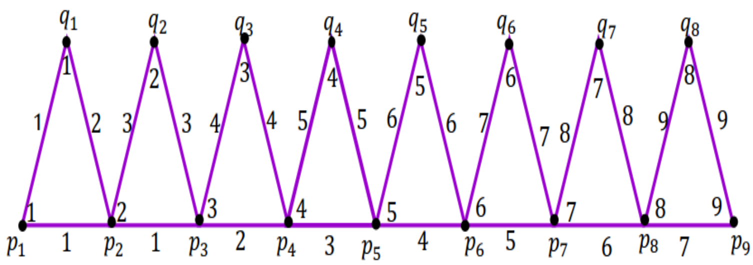

Figure (12) The triangular snake graph T_9

3. Conclusion

In this paper, we determined the exact value of TEIS for half gear graph HG_m, double fan graph $F_{2,m}$, and triangular snake graph T_m as shown in the following formulas:

$$\begin{aligned} tes(HG_m) &= m, \\ tes(F_{2,m}) &= m+1, \\ tes(T_m) &= m. \end{aligned}$$

Competing interests

The authors declare that they have no competing interests.

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Author's contributions

All authors equally contributed to this work. All authors read and approved the final manuscript.

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