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A CONCEPTUAL FRAMEWORK OF m-CONVEX AND m-CONCAVE SETS UNDER SOFT SET ENVIRONMENT WITH PROPERTIES

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Abstract. In this paper, the classical notions of m-convex and m-concave sets are characterized under soft set environment and their important aggregation operations are discussed. Moreover, certain classical approaches (i.e. first and second senses) are employed on m-convex and m-concave soft sets to get more generalized results for uncertain scenarios.
Keywords: Soft set, Convex soft set, Concave soft set.

1. Introduction

In 1999, Molodtsov [1] introduced the theory of soft sets in literature as a new parameterized family of subsets of the universe of discourse. In 2003, Maji et al. [2] extended the concept and introduced some fundamental terminologies and operations which later on further extended by Pei et al. [4] in 2005, Ali et al. [3] in 2009 and Babitha et al. [5, 6] in 2010, 2011 for set operations, the restrictions on different set theoretic operations and soft set relations respectively. Ge et al. [7] and Çağman et al. [8, 9] and gave some modifications in the work of Maji et al. [2] and also established some new results. Deli [10, 11] defined convex and concave sets under soft set and fuzzy soft set settings. He discussed some of their important results. In 2020, Rahman et al. [14] conceptualized convexity cum concavity on hypersoft set (i.e. an extension of soft set) and presented its pictorial versions with illustrative examples. In this work, we extend the concept of convexity on soft sets defined in [10] and develop m-convex and m-concave soft sets with their generalizations in first and second senses. The pattern for rest of the paper is as under:
Section 2: reviews fundamental definitions and terms from literature.
Section 3: presents the novel concept of m-convex and m-concave soft sets
Section 4: describes the new concept of m-convex and m-concave soft sets in first and second senses.
Section 5: concludes the paper.

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2. PRELIMINARIES

In this section, some fundamental terms and results of soft set theory are presented. For more detail, please see ([1],[2],[9]). Throughout the paper, $\check{L}$, $\check{U}$ and $\check{J}$ will play the role of $\mathbb{R}^n$, arbitrary set (universe of discourse) and $[0,1]$ respectively.

Definition 2.1. *[1] Let $\check{P}(\check{U})$ be the power set of $\check{U}$ (universe of discourse) and $\check{L}$ be a set of parameters defining $\check{U}$. A* soft set *$\check{M}$ over $\check{U}$ is a set defined by a set valued function $\check{M}$ representing a mapping*

$$\check{h}_{\check{M}} : \check{L} \to P(\check{U})$$

Definition 2.2. *[8] If $\check{h}_{\check{M}}(\omega) \subseteq \check{h}_{\check{N}}(\omega)$ for all $\omega \in \check{L}$, then $\check{M}$ is a soft subset of $\check{N}$, denoted by $\check{M}\widetilde{\subseteq}\check{N}$*

Definition 2.3. *[8] Union of sets $\check{M}$ and $\check{N}$ ($M\widetilde{U}N$) is defined as*

$$\check{h}_{\check{M}\widetilde{U}\check{N}}(\omega) = \check{h}_{\check{M}}(\omega) \cup \check{h}_{\check{N}}(\omega) \forall \omega \in \check{L}$$

Definition 2.4. *[8] Intersection of set $\check{M}$ and $\check{N}$ ($\check{M}\widetilde{\cap}\check{N}$) is defined as*

$$\check{h}_{\check{M}\widetilde{\cap}\check{N}}(\omega) = \check{h}_{\check{M}}(\omega) \cap \check{h}_{\check{N}}(\omega) \forall \omega \in \check{L}$$

Definition 2.5. *[9] The $\check{\delta}-$inclusion of a soft set $\check{M}$ (where $\check{\delta}\widetilde{\subseteq}\check{U}$) is defined by*

$$\check{M}^{\check{\delta}} = \left\{ \omega \in \check{L} : \check{h}_{\check{M}}(\omega) \supseteq \check{\delta} \right\}$$

Definition 2.6. *[10]*
The soft set $\check{M}$ on $\check{L}$ is called a convex soft set if

$$\check{h}_{\check{M}}(\varepsilon\omega + (1-\varepsilon)\mu) \supseteq \check{h}_{\check{M}}(\omega) \cap \check{h}_{\check{M}}(\mu)$$

for every $\omega, \mu \in \check{L}$ and $\varepsilon \in \check{J}$.

Definition 2.7. *[10]*
The soft set $\check{M}$ on $\check{L}$ is called a concave soft set if

$$\check{h}_{\check{M}}(\varepsilon\omega + (1-\varepsilon)\mu) \subseteq \check{h}_{\check{M}}(\omega) \cup \check{h}_{\check{M}}(\mu)$$

for every $\omega, \mu \in \check{L}$ and $\varepsilon \in \check{J}$.

3. *m*-CONVEX AND *m*-CONCAVE SOFT SETS

In this section, *m*-convex and *m*-concave soft sets are defined and then some desired results are proved. Some classical definitions are followed from [12, 13].

Definition 3.1. *The soft set $\check{M}$ on $\check{L}$ is called m-convex soft set if*

$$\check{h}_{\check{M}}(n\omega_1 + m(1-n)\omega_2) \supseteq \check{h}_{\check{M}}(\omega_1) \cap \check{h}_{\check{M}}(\omega_2) \tag{1}$$

for ω_1, $\omega_2 \in \check{L}$, $m \in \check{J}$ and $n \in (0,1]$.

Theorem 3.2. *$\check{M} \cap \check{N}$ is m-convex soft set when both $\check{M}$ and $\check{N}$ are m-convex soft sets.*

Proof. Suppose that for $\omega_1, \omega_2 \in \check{L}$, and $\check{W} = \check{M} \cap \check{N}$, Then,

$$\check{h}_{\check{W}}(n\omega_1 + m(1-n)\omega_2) = \check{h}_{\check{M}}(n\omega_1 + m(1-n)\omega_2) \cap \check{h}_{\check{N}}(n\omega_1 + m(1-n)\omega_2) \tag{2}$$

As $\check{M}$ and $\check{N}$ are *m*-convex,

$$\check{h}_{\check{M}}(n\omega_1 + m(1-n)\omega_2) \supseteq \check{h}_{\check{M}}(\omega_1) \cap \check{h}_{\check{M}}(\omega_2) \tag{3}$$

$$\check{h}_{\check{N}}(n\omega_1 + m(1-n)\omega_2) \supseteq \check{h}_{\check{N}}(\omega_1) \cap \check{h}_{\check{N}}(\omega_2) \tag{4}$$

which implies

$$\check{h}_{\check{W}}(n\omega_1 + m(1-n)\omega_2) \supseteq (\check{h}_{\check{M}}(\omega_1) \cap \check{h}_{\check{M}}(\omega_2)) \cap (\check{h}_{\check{N}}(\omega_1) \cap \check{h}_{\check{N}}(\omega_2)) \tag{5}$$

and thus

$$\check{h}_{\check{W}}(n\omega_1 + m(1-n)\omega_2) \supseteq \check{h}_{\check{W}}(\omega) \cap \check{h}_{\check{W}}(\omega_2) \tag{6}$$

□

Theorem 3.3. *$\check{M}$ is m-convex soft set on $\check{L}$ iff for every $n \in [0,1]$ and $\check{\delta} \in \check{P}(\check{U})$, $\check{M}^{\check{\delta}}$ is m-convex set on $\check{L}$.*

Proof. Suppose $\check{M}$ is *m*-convex soft set. If $\omega_1, \omega_2 \in \check{L}$ and $\check{\delta} \in \check{P}(\check{U})$, then $\check{h}_{\check{M}}(\omega_1) \supseteq \check{\delta}$ and $\check{h}_{\check{M}}(\omega_2) \supseteq \check{\delta}$.
$\Rightarrow \check{h}_{\check{M}}(\omega_1) \cap \check{h}_{\check{M}}(\omega_2) \supseteq \check{\delta}$
$\Rightarrow \check{h}_{\check{M}}(n\omega_1 + m(1-n)\omega_2) \supseteq \check{h}_{\check{M}}(\omega_1) \cap \check{h}_{\check{M}}(\omega_2) \supseteq \check{\delta}$
$\Rightarrow \check{h}_{\check{M}}(n\omega_1 + m(1-n)\omega_2) \supseteq \check{\delta}$
and thus $\check{M}^{\check{\delta}}$ is *m*-convex set.
Conversely suppose that $\check{M}^{\check{\delta}}$ is *m*-convex set for every $n \in [0,1]$.For $\omega_1, \omega_2 \in \check{L}$, $\check{M}^{\check{\delta}}$ is *m*-convex with $\check{\delta} = \check{h}_{\check{M}}(\omega_1) \cap \check{h}_{\check{M}}(\omega_2)$. Since $\check{h}_{\check{M}}(\omega_1) \supseteq \check{\delta}$ and $\check{h}_{\check{M}}(\omega_2) \supseteq \check{\delta}$, we have $\omega_1 \in \check{M}^{\check{\delta}}$ and $\omega_2 \in \check{M}^{\check{\delta}}$,
$\Rightarrow n\omega_1 + m(1-n)\omega_2 \in \check{M}^{\check{\delta}}$.
Therefore,

$$\check{h}_{\check{M}}(n\omega_1 + m(1-n)\omega_2) \supseteq \check{h}_{\check{M}}(\omega_1) \cap \check{h}_{\check{M}}(\omega_2),$$

which proves the *m*-convexity of $\check{M}$ on $\check{L}$. □

Definition 3.4. *The soft set $\check{M}$ on $\check{L}$ is called m-concave soft set if*

$$\check{h}_{\check{M}}(n\omega_1 + m(1-n)\omega_2) \subseteq \check{h}_{\check{M}}(\omega_1) \cup \check{h}_{\check{M}}(\omega_2) \tag{7}$$

for $\omega_1, \omega_2 \in \check{L}$, $m \in \check{J}$ and $n \in (0,1]$.

Theorem 3.5. *$\check{M} \cup \check{N}$ is m-concave soft set when both $\check{M}$ and $\check{N}$ are m-concave soft sets.*

Proof. Suppose that for $\omega_1, \omega_2 \in \check{L}$, and $n \in \check{J}$ and $\check{W} = \check{M} \cup \check{N}$. Then,

$$\check{h}_{\check{W}}(n\omega_1 + m(1-n)\omega_2) = \check{h}_{\check{M}}(n\omega_1 + m(1-n)\omega_2) \cup \check{h}_{\check{N}}(n\omega_1 + m(1-n)\omega_2) \tag{8}$$

Now, since $\check{M}$ and $\check{N}$ are *m*-concave,

$$\check{h}_{\check{M}}(n\omega_1 + m(1-n)\omega_2) \subseteq \check{h}_{\check{M}}(\omega_1) \cup \check{h}_{\check{M}}(\omega_2) \tag{9}$$

$$\check{h}_{\check{N}}(n\omega_1 + m(1-n)\omega_2) \subseteq \check{h}_{\check{N}}(\omega_1) \cup \check{h}_{\check{N}}(\omega_2) \tag{10}$$

and hence,

$$\check{h}_{\check{W}}(n\omega_1 + m(1-n)\omega_2) \subseteq \left(\check{h}_{\check{M}}(\omega_1) \cup \check{h}_{\check{M}}(\omega_2)\right) \cup \left(\check{h}_{\check{N}}(\omega_1) \cup \check{h}_{\check{N}}(\omega_2)\right) \tag{11}$$

and thus

$$\check{h}_{\check{W}}(n\omega_1 + m(1-n)\omega_2) \subseteq \check{h}_{\check{W}}(\omega_1) \cup \check{h}_{\check{W}}(\omega_2) \tag{12}$$

□

Theorem 3.6. *$\check{M} \cap \check{N}$ is m-concave soft set when both $\check{M}$ and $\check{N}$ are m-concave soft sets.*

Proof. Suppose that for $\omega_1, \omega_2 \in \check{L}$, and $n \in \check{J}$ and $\check{W} = \check{M} \cap \check{N}$. Then,

$$\check{h}_{\check{W}}(n\omega_1 + m(1-n)\omega_2) = \check{h}_{\check{M}}(n\omega_1 + m(1-n)\omega_2) \cap \check{h}_{\check{N}}(n\omega_1 + m(1-n)\omega_2) \tag{13}$$

Now, since $\check{M}$ and $\check{N}$ are *m*-concave,

$$\check{h}_{\check{M}}(n\omega_1 + m(1-n)\omega_2) \subseteq \check{h}_{\check{M}}(\omega_1) \cup \check{h}_{\check{M}}(\omega_2) \tag{14}$$

$$\check{h}_{\check{N}}(n\omega_1 + m(1-n)\omega_2) \subseteq \check{h}_{\check{N}}(\omega_1) \cup \check{h}_{\check{N}}(\omega_2) \tag{15}$$

and hence,

$$\check{h}_{\check{W}}(n\omega_1 + m(1-n)\omega_2) \subseteq \left(\check{h}_{\check{M}}(\omega_1) \cup \check{h}_{\check{M}}(\omega_2)\right) \cap \left(\check{h}_{\check{N}}(\omega_1) \cup \check{h}_{\check{N}}(\omega_2)\right) \tag{16}$$

and thus

$$\check{h}_{\check{W}}(n\omega_1 + m(1-n)\omega_2) \subseteq \check{h}_{\check{W}}(\omega_1) \cup \check{h}_{\check{W}}(\omega_2) \tag{17}$$

□

Theorem 3.7. *$\check{M}^c$ is m-concave soft set when $\check{M}$ is m-convex soft set.*

Proof. Suppose that for $\omega_1, \omega_2 \in \check{L}$, $n \in \check{J}$, and $\check{M}$ be *m*-convex soft set.
Since $\check{M}$ is *m*-convex,

$$\check{h}_{\check{M}}(n\omega_1 + m(1-n)\omega_2) \supseteq \check{h}_{\check{M}}(\omega_1) \cap \check{h}_{\check{M}}(\omega_2) \tag{18}$$

or

$$\check{U} \setminus \check{h}_{\check{M}}(n\omega_1 + m(1-n)\omega_2) \subseteq \check{U} \setminus \left\{\check{h}_{\check{M}}(\omega_1) \cap \check{h}_{\check{M}}(\omega_2)\right\} \tag{19}$$

If $\check{h}_{\check{M}}(\omega_1) \supset \check{h}_{\check{M}}(\omega_2)$ then $\check{h}_{\check{M}}(\omega_1) \cap \check{h}_{\check{M}}(\omega_2) = \check{h}_{\check{M}}(\omega_2)$
then,

$$\check{U} \setminus \check{h}_{\check{M}}(n\omega_1 + m(1-n)\omega_2) \subseteq \check{U} \setminus \check{h}_{\check{M}}(\omega_2)$$

If $\check{h}_{\check{M}}(\omega_1) \subset \check{h}_{\check{M}}(\omega_2)$ then $\check{h}_{\check{M}}(\omega_1) \cap \check{h}_{\check{M}}(\omega_2) = \check{h}_{\check{M}}(\omega_1)$
then we may write

$$\check{U} \setminus \check{h}_{\check{M}}(n\omega_1 + m(1-n)\omega_2) \subseteq \check{U} \setminus \check{h}_{\check{M}}(\omega_1) \tag{20}$$

so we have

$$\check{U} \setminus \check{h}_{\check{M}}(n\omega_1 + m(1-n)\omega_2) \subseteq \left\{\check{U} \setminus \check{h}_{\check{M}}(\omega_1) \cup \check{U} \setminus \check{h}_{\check{M}}(\omega_2)\right\}. \tag{21}$$

which shows that $\check{M}^c$ is *m*-concave soft set. □

Theorem 3.8. *$\check{M}^c$ is m-convex soft set when $\check{M}$ is m-concave soft set.*

Proof. Suppose that for $\omega_1, \omega_2 \in \check{L}$, $n \in \check{J}$, and $\check{M}$ be m-concave soft set.
Since $\check{M}$ is m-concave,

$$\check{h}_{\check{M}}(n\omega_1 + m(1-n)\omega_2) \subseteq \check{h}_{\check{M}}(\omega_1) \cup \check{h}_{\check{M}}(\omega_2) \tag{22}$$

or

$$\check{U} \backslash \check{h}_{\check{M}}(n\omega_1 + m(1-n)\omega_2) \supseteq \check{U} \backslash \left\{\check{h}_{\check{M}}(\omega_1) \cup \check{h}_{\check{M}}(\omega_2)\right\} \tag{23}$$

If $\check{h}_{\check{M}}(\omega_1) \supset \check{h}_{\check{M}}(\omega_2)$ then $\check{h}_{\check{M}}(\omega_1) \cup \check{h}_{\check{M}}(\omega_2) = \check{h}_{\check{M}}(\omega_1)$
then,

$$\check{U} \backslash \check{h}_{\check{M}}(n\omega_1 + m(1-n)\omega_2) \supseteq \check{U} \backslash \check{h}_{\check{M}}(\omega_1)$$

If $\check{h}_{\check{M}}(\omega_1) \subset \check{h}_{\check{M}}(\omega_2)$ then $\check{h}_{\check{M}}(\omega_1) \cup \check{h}_{\check{M}}(\omega_2) = \check{h}_{\check{M}}(\omega_2)$
then we may write

$$\check{U} \backslash \check{h}_{\check{M}}(n\omega_1 + m(1-n)\omega_2) \supseteq \check{U} \backslash \check{h}_{\check{M}}(\omega_2) \tag{24}$$

so we have

$$\check{U} \backslash \check{h}_{\check{M}}(n\omega_1 + m(1-n)\omega_2) \supseteq \left\{\check{U} \backslash \check{h}_{\check{M}}(\omega_1) \cap \check{U} \backslash \check{h}_{\check{M}}(\omega_2)\right\}. \tag{25}$$

So, $\check{M}^c$ is m-convex soft set. □

Theorem 3.9. *$\check{M}$ is m-concave soft set on $\check{L}$ iff for every $n \in [0,1]$ and $\check{\delta} \in \check{P}(\check{U})$,
$\check{M}^{\check{\delta}}$ is m-concave set on $\check{L}$.*

Proof. Suppose that $\check{M}$ is m-concave soft set.If $\omega_1, \omega_2 \in \check{L}$ and $\check{\delta} \in \check{P}(\check{U})$, then $\check{h}_{\check{M}}(\omega_1) \supseteq \check{\delta}$ and $\check{h}_{\check{M}}(\omega_2) \supseteq \check{\delta}$ then $\check{h}_{\check{M}}(\omega_1) \cup \check{h}_{\check{M}}(\omega_2) \supseteq \check{\delta}$. It follows from m-concavity of $\check{M}$ that

$$\check{\delta} \subseteq \check{h}_{\check{M}}(\omega_1) \cap \check{h}_{\check{M}}(\omega_2) \subseteq \check{h}_{\check{M}}(n\omega_1 + m(1-n)\omega_2) \subseteq \check{h}_{\check{M}}(\omega_1) \cup \check{h}_{\check{M}}(\omega_2)$$

$\Rightarrow$ $\check{\delta} \subseteq \check{h}_{\check{M}}(n\omega_1 + m(1-n)\omega_2)$
$\Rightarrow$ $\check{M}^{\check{\delta}}$ is a concave set.
Conversely suppose that $\check{M}^{\check{\delta}}$ is m-concave set for every $n \in [0,1]$. For $\omega_1, \omega_2 \in \check{L}$, $\check{M}^{\check{\delta}}$ is concave with $\check{\delta} = \check{h}_{\check{M}}(\omega_1) \cup \check{h}_{\check{M}}(\omega_2)$. Since $\check{h}_{\check{M}}(\omega_1) \subseteq \check{\delta}$ and $\check{h}_{\check{M}}(\omega_2) \subseteq \check{\delta}$, we have $\omega_1 \in \check{M}^{\check{\delta}}$ and $\omega_2 \in \check{M}^{\check{\delta}}$, hence $n\omega_1 + m(1-n)\omega_2 \in \check{M}^{\check{\delta}}$.
$\Rightarrow$ $\check{h}_{\check{M}}(n\omega_1 + m(1-n)\omega_2) \subseteq \check{M}^{\check{\delta}}$
Therefore, $\check{h}_{\check{M}}(n\omega_1 + m(1-n)\omega_2) \subseteq \check{h}_{\check{M}}(\omega_1) \cup \check{h}_{\check{M}}(\omega_2)$, which proves the m-concavity of $\check{M}$ on $\check{X}$. □

4. m-CONVEX AND m-CONCAVE SOFT SETS IN FIRST AND SECOND SENSES

In this section, m-convex and m-concave soft sets are defined in first and second senses and then some essential results are proved.

Definition 4.1. *The soft set $\check{M}$ on $\check{L}$ is called a m-convex soft set in first sense if*

$$\check{h}_{\check{M}}(n\omega_1 + m(1-n^{\eta})\omega_2) \tilde{\supseteq} \check{h}_{\check{M}}(\omega_1) \tilde{\cap} \check{h}_{\check{M}}(\omega_2) \tag{26}$$

for $\omega_1, \omega_2 \in \check{L}$, $m \in \check{J}$ and $\eta, n \in (0,1]$.

Definition 4.2. *The soft set $\check{M}$ on $\check{L}$ is called a m-convex soft set in a second sense if*

$$\check{h}_{\check{M}}(n\omega_1 + m(1-n)^{\eta}\omega_2) \tilde{\supseteq} \check{h}_{\check{M}}(\omega_1) \tilde{\cap} \check{h}_{\check{M}}(\omega_2) \tag{27}$$

for ω_1, $\omega_2 \in \check{L}$, $m \in \check{J}$ *and* $\eta, n \in (0,1]$.

Theorem 4.3. *$\check{M}\tilde{\cap}\check{N}$ is a m-convex soft set when both $\check{M}$ and $\check{N}$ are m-convex soft sets in the first sense.*

Proof. Suppose that for ω_1, $\omega_2 \in \check{L}$, and $\check{W} = \check{M}\tilde{\cap}\check{N}$, Then,

$$\check{h}_{\check{W}}(n\omega_1 + m(1-n^{\eta})\omega_2) = \check{h}_{\check{M}}(n\omega_1 + m(1-n^{\eta})\omega_2) \tilde{\cap} \check{h}_{\check{N}}(n\omega_1 + m(1-n^{\eta})\omega_2) \tag{28}$$

Now, since $\check{M}$ and $\check{N}$ are *m*-convex in the first sense,

$$\check{h}_{\check{M}}(n\omega_1 + m(1-n^{\eta})\omega_2) \tilde{\supseteq} \check{h}_{\check{M}}(\omega_1) \tilde{\cap} \check{h}_{\check{M}}(\omega_2) \tag{29}$$

$$\check{h}_{\check{N}}(n\omega_1 + m(1-n^{\eta})\omega_2) \tilde{\supseteq} \check{h}_{\check{N}}(\omega_1) \tilde{\cap} \check{h}_{\check{N}}(\omega_2) \tag{30}$$

which implies

$$\check{h}_{\check{W}}(n\omega_1 + m(1-n^{\eta})\omega_2) \tilde{\supseteq} (\check{h}_{\check{M}}(\omega_1) \tilde{\cap} \check{h}_{\check{M}}(\omega_2)) \tilde{\cap} (\check{h}_{\check{N}}(\omega_1) \tilde{\cap} \check{h}_{\check{N}}(\omega_2)) \tag{31}$$

and thus

$$\check{h}_{\check{W}}(n\omega_1 + m(1-n^{\eta})\omega_2) \tilde{\supseteq} \check{h}_{\check{W}}(\omega) \tilde{\cap} \check{h}_{\check{W}}(\omega_2) \tag{32}$$

□

Theorem 4.4. *$\check{M}\tilde{\cap}\check{N}$ is a m-convex soft set in the second sense when both $\check{M}$ and $\check{N}$ are m-convex soft sets in the second sense.*

Proof. Suppose that for $\omega_1, \omega_2 \in \check{L}$, and $\check{W} = \check{M}\tilde{\cap}\check{N}$, Then,

$$\check{h}_{\check{W}}(n\omega_1 + m(1-n)^{\eta}\omega_2) = \check{h}_{\check{M}}(n\omega_1 + m(1-n)^{\eta}\omega_2) \tilde{\cap} \check{h}_{\check{N}}(n\omega_1 + m(1-n)^{\eta}\omega_2) \tag{33}$$

Now, since $\check{M}$ and $\check{N}$ are *m*-convex in the second sense,

$$\check{h}_{\check{M}}(n\omega_1 + m(1-n)^{\eta}\omega_2) \tilde{\supseteq} \check{h}_{\check{M}}(\omega_1) \tilde{\cap} \check{h}_{\check{M}}(\omega_2) \tag{34}$$

$$\check{h}_{\check{N}}(n\omega_1 + m(1-n)^{\eta}\omega_2) \tilde{\supseteq} \check{h}_{\check{N}}(\omega_1) \tilde{\cap} \check{h}_{\check{N}}(\omega_2) \tag{35}$$

which implies

$$\check{h}_{\check{W}}(n\omega_1 + m(1-n)^{\eta}\omega_2) \tilde{\supseteq} (\check{h}_{\check{M}}(\omega_1) \tilde{\cap} \check{h}_{\check{M}}(\omega_2)) \tilde{\cap} (\check{h}_{\check{N}}(\omega_1) \tilde{\cap} \check{h}_{\check{N}}(\omega_2)) \tag{36}$$

and thus

$$\check{h}_{\check{W}}(n\omega_1 + m(1-n)^{\eta}\omega_2) \tilde{\supseteq} \check{h}_{\check{W}}(\omega) \tilde{\cap} \check{h}_{\check{W}}(\omega_2) \tag{37}$$

□

Theorem 4.5. *$\breve{M}$ is a m-convex soft set in the first sense on $\breve{L}$ iff for every $n \in [0,1]$ and $\breve{\delta} \in \breve{P}(\breve{U})$, $\breve{M}^{\breve{\delta}}$ is a m-convex set in the first sense on $\breve{L}$.*

Proof. Suppose $\breve{M}$ is a m-convex soft set in the first sense. If $\omega_1, \omega_2 \in \breve{L}$, and $\breve{\delta} \in \breve{P}(\breve{U})$, then $\breve{h}_{\breve{M}}(\omega_1) \tilde{\supseteq} \breve{\delta}$ and $\breve{h}_{\breve{M}}(\omega_2) \tilde{\supseteq} \breve{\delta}$. It follows from m-convexity of $\breve{M}$ that

$$\breve{h}_{\breve{M}}(n\omega_1 + m(1-n^{\eta})\omega_2) \tilde{\supseteq} \breve{h}_{\breve{M}}(\omega_1) \tilde{\cap} \breve{h}_{\breve{M}}(\omega_2) \tag{38}$$

and thus $M^{\breve{\delta}}$ is a m-convex set in the first sense.
Conversely suppose that $\breve{M}^{\breve{\delta}}$ is a m-convex set in the first sense for every $n \in [0,1]$. For $\omega_1, \omega_2 \in \breve{L}$, $\breve{M}^{\breve{\delta}}$ is m-convex for $\breve{\delta} = \breve{h}_{\breve{M}}(\omega_1) \tilde{\cap} \breve{h}_{\breve{M}}(\omega_2)$. Since $\breve{h}_{\breve{M}}(\omega_1) \tilde{\supseteq} \breve{\delta}$ and $\breve{h}_{\breve{M}}(\omega_2) \tilde{\supseteq} \breve{\delta}$, we have $\omega_1 \in \breve{M}^{\breve{\delta}}$ and $\omega_2 \in \breve{M}^{\breve{\delta}}$, hence $n\omega_1 + m(1-n^{\eta})\omega_2 \in \breve{M}^{\breve{\delta}}$.Therefore,
$\breve{h}_{\breve{M}}(n\omega_1 + m(1-n^{\eta})\omega_2) \tilde{\supseteq} \breve{h}_{\breve{M}}(\omega_1) \tilde{\cap} \breve{h}_{\breve{M}}(\omega_2)$, which indicates S is a m-convex set in the first sense on X. □

Theorem 4.6. *$\breve{M}$ is a m-convex soft set in the second sense on $\breve{L}$ iff for every $n \in [0,1]$ and $\breve{\delta} \in \breve{P}(\breve{U})$, $\breve{M}^{\breve{\delta}}$ is a m-convex set in the second sense on $\breve{L}$.*

Proof. Suppose $\breve{M}$ is a m-convex soft set in the second sense. If $\omega_1, \omega_2 \in \breve{L}$ and $\breve{\delta} \in \breve{P}(\breve{U})$,then $\breve{h}_{\breve{M}}(\omega_1) \tilde{\supseteq} \breve{\delta}$ and $\breve{h}_{\breve{M}}(\omega_2) \tilde{\supseteq} \breve{\delta}$. It follows from the m-convexity of $\breve{M}$ that

$$\breve{h}_{\breve{M}}\left(n\omega_1 + m(1-n)^{\eta}\omega_2\right) \tilde{\supseteq} \breve{h}_{\breve{M}}(\omega_1) \tilde{\cap} \breve{h}_{\breve{M}}(\omega_2) \tag{39}$$

and thus $\breve{M}^{\breve{\delta}}$ is a m-convex set in the second sense.
Conversly suppose that $\breve{M}^{\breve{\delta}}$ is a m-convex set in the second sense for every $\rho \in [0,1]$. For $\omega_1, \omega_2 \in \breve{L}$, $\breve{M}^{\breve{\delta}}$ is m-convex for $\breve{\delta} = \breve{h}_{\breve{M}}(\omega_1) \tilde{\cap} \breve{h}_{\breve{M}}(\omega_2)$. Since $\breve{h}_{\breve{M}}(\omega_1) \tilde{\supseteq} \breve{\delta}$ and $\breve{h}_{\breve{M}}(\omega_2) \tilde{\supseteq} \breve{\delta}$, we have $\omega_1 \in \breve{M}^{\breve{\delta}}$ and $\omega_2 \in M^{\breve{\delta}}$, hence $n\omega_1 + m(1-n)^{\eta}\omega_2 \in \breve{M}^{\breve{\delta}}$. Therefore,
$\breve{h}_{\breve{M}}\left(n\omega_1 + m(1-n)^{\eta}\omega_2\right) \tilde{\supseteq} \breve{h}_{\breve{M}}(\omega_1) \tilde{\cap} \breve{h}_{\breve{M}}(\omega_2)$, which indicates $\breve{M}$ is a m-convex set in the second sense on $\breve{X}$. □

Definition 4.7. *The soft set$\breve{M}$ on $\breve{L}$ is called a m-concave soft set in the first sense if*

$$\breve{h}_{\breve{M}}(n\omega_1 + m(1-n^{\eta})\omega_2) \tilde{\subseteq} \breve{h}_{\breve{M}}(\omega_1) \tilde{\cup} \breve{h}_{\breve{M}}(\omega_2) \tag{40}$$

for every $\omega_1, \omega_2 \in \breve{L}$, $m \in \breve{J}$ and $\eta, n \in (0,1]$.

Definition 4.8. *The soft set $\breve{M}$ on $\breve{L}$ is called a m-concave soft set in the second sense if*

$$\breve{h}_{\breve{M}}(n\omega_1 + m(1-n)^{\eta}\omega_2) \tilde{\subseteq} \breve{h}_{\breve{M}}(\omega_1) \tilde{\cup} \breve{h}_{\breve{M}}(\omega_2) \tag{41}$$

for every $\omega_1, \omega_2 \in \breve{L}$, $m \in \breve{J}$ and $\eta, n \in (0,1]$.

Theorem 4.9. *$\breve{M} \tilde{\cap} \breve{N}$ is a m-concave soft set in the first sense when both $\breve{M}$ and $\breve{N}$ are m-concave soft sets in the first sense.*

Proof. Suppose that for $\omega_1, \omega_2 \in \check{L}$, and $n \in \check{J}$ and $\check{W} = \check{M} \tilde{\cap} \check{N}$. Then,

$$\check{h}_{\check{W}}(n\omega_1 + m(1-n^{\eta})\omega_2) = \check{h}_{\check{M}}(n\omega_1 + m(1-n^{\eta})\omega_2) \tilde{\cap} \check{h}_{\check{N}}(n\omega_1 + m(1-n^{\eta})\omega_2) \tag{42}$$

Now, since S and T are *m*-concave in the first sense,

$$\check{h}_{\check{M}}(n\omega_1 + m(1-n^{\eta})\omega_2) \tilde{\subseteq} \check{h}_{\check{M}}(\omega_1) \tilde{\cup} \check{h}_{\check{M}}(\omega_2) \tag{43}$$

$$\check{h}_{\check{N}}(n\omega_1 + m(1-n^{\eta})\omega_2) \tilde{\subseteq} \check{h}_{\check{N}}(\omega_1) \tilde{\cup} \check{h}_{\check{N}}(\omega_2) \tag{44}$$

and hence,

$$\check{h}_{\check{W}}(n\omega_1 + m(1-n^{\eta})\omega_2) \tilde{\subseteq} \left(\check{h}_{\check{M}}(\omega_1) \tilde{\cup} \check{h}_{\check{M}}(\omega_2)\right) \tilde{\cap} \left(\check{h}_{\check{N}}(\omega_1) \tilde{\cup} \check{h}_{\check{N}}(\omega_2)\right) \tag{45}$$

and thus

$$\check{h}_{\check{W}}(n\omega_1 + m(1-n^{\eta})\omega_2) \tilde{\subseteq} \check{h}_{\check{W}}(\omega_1) \tilde{\cup} \check{h}_{\check{W}}(\omega_2) \tag{46}$$

□

Theorem 4.10. *$\check{M} \tilde{\cap} \check{N}$ is a m-concave soft set in the second sense when both $\check{M}$ and $\check{N}$ are m-concave soft sets in the second sense.*

Proof. Suppose that for $\omega_1, \omega_2 \in \check{L}$, and $n \in \check{J}$ and $\check{W} = \check{M} \tilde{\cap} \check{N}$. Then,

$$\check{h}_{\check{W}}(n\omega_1 + m(1-n)^{\eta}\omega_2) = \check{h}_{\check{M}}(n\omega_1 + m(1-n)^{\eta}\omega_2) \tilde{\cap} \check{h}_{\check{N}}(n\omega_1 + m(1-n)^{\eta}\omega_2) \tag{47}$$

Now, since $\check{M}$ and $\check{N}$ are *m*-concave in the second sense,

$$\check{h}_{\check{M}}(n\omega_1 + m(1-n)^{\eta}\omega_2) \tilde{\subseteq} \check{h}_{\check{M}}(\omega_1) \tilde{\cup} \check{h}_{\check{M}}(\omega_2) \tag{48}$$

$$\check{h}_{\check{N}}(n\omega_1 + m(1-n)^{\eta}\omega_2) \tilde{\subseteq} \check{h}_{\check{N}}(\omega_1) \tilde{\cup} \check{h}_{\check{N}}(\omega_2) \tag{49}$$

and hence,

$$\check{h}_{\check{W}}(n\omega_1 + m(1-n)^{\eta}\omega_2) \tilde{\subseteq} \left(\check{h}_{\check{M}}(\omega_1) \tilde{\cup} \check{h}_{\check{M}}(\omega_2)\right) \tilde{\cap} \left(\check{h}_{\check{N}}(\omega_1) \tilde{\cup} \check{h}_{\check{N}}(\omega_2)\right) \tag{50}$$

and thus

$$\check{h}_{\check{W}}(n\omega_1 + m(1-n)^{\eta}\omega_2) \tilde{\subseteq} \check{h}_{\check{W}}(\omega_1) \tilde{\cup} \check{h}_{\check{W}}(\omega_2) \tag{51}$$

□

Theorem 4.11. *$\check{M}^c$ is a m-concave soft set in the second sense when $\check{M}$ is a m-convex soft set in the second sense.*

Proof. Suppose that for $\omega_1, \omega_2 \in \check{L}$, $n \in \check{J}$, and $\check{M}$ be a *m*-convex soft set in the second sense.
then, since $\check{M}$ is *m*-convex in the second sense,

$$\check{h}_{\check{M}}(n\omega_1 + m(1-n)^{\eta}\omega_2) \tilde{\supseteq} \check{h}_{\check{M}}(\omega_1) \tilde{\cap} \check{h}_{\check{M}}(\omega_2) \tag{52}$$

or

$$\check{U} \setminus \check{h}_{\check{M}}(n\omega_1 + m(1-n)^{\eta}\omega_2) \tilde{\subseteq} \check{U} \setminus \left\{\check{h}_{\check{M}}(\omega_1) \tilde{\cap} \check{h}_{\check{M}}(\omega_2)\right\} \tag{53}$$

If $\check{h}_{\check{M}}(\omega_1) \supset \check{h}_{\check{M}}(\omega_2)$ then we may write

$$\check{U} \setminus \check{h}_{\check{M}}(n\omega_1 + m(1-n)^{\eta}\omega_2) \tilde{\subseteq} \check{U} \setminus \check{h}_{\check{M}}(\omega_2) \tag{54}$$

If $\breve{h}_{\breve{M}}(\omega_1) \subset \breve{h}_{\breve{M}}(\omega_2)$ then we may write

$$\breve{U} \setminus \breve{h}_{\breve{M}}(n\omega_1 + m(1-n)^{\eta}\omega_2) \tilde{\subseteq} \breve{U} \setminus \breve{h}_{\breve{M}}(\omega_1) \tag{55}$$

From the above equations, we have

$$\breve{U} \setminus \breve{h}_{\breve{M}}(n\omega_1 + m(1-n)^{\eta}\omega_2) \tilde{\subseteq} (\breve{U} \setminus \breve{h}_{\breve{M}}(\omega_1)) \tilde{\cup} (\breve{U} \setminus \breve{h}_{\breve{M}}(\omega_2)). \tag{56}$$

So, $\breve{M}^c$ is a m-concave soft set in the second sense. □

Theorem 4.12. *$\breve{M}^c$ is a m-convex soft set in the first sense when $\breve{M}$ is a m-concave soft set in the first sense.*

Proof. Suppose that there exist $\omega_1, \omega_2 \in \breve{L}$, $n \in \breve{J}$ and $\breve{M}$ be a m-concave soft set in the first sense.
then, since $\breve{M}$ is m-concave in the first sense,

$$\breve{h}_{\breve{M}}(n\omega_1 + m(1-n^{\eta})\omega_2) \tilde{\subseteq} \breve{h}_{\breve{M}}(\omega_1) \tilde{\cup} \breve{h}_{\breve{M}}(\omega_2) \tag{57}$$

or

$$\breve{U} \setminus \breve{h}_{\breve{M}}(n\omega_1 + m(1-n^{\eta})\omega_2) \tilde{\supseteq} \breve{U} \setminus \left\{\breve{h}_{\breve{M}}(\omega_1) \tilde{\cup} \breve{h}_{\breve{M}}(\omega_2)\right\} \tag{58}$$

If $\breve{h}_{\breve{M}}(\omega_1) \supset \breve{h}_{\breve{M}}(\omega_2)$ then we may write

$$\breve{U} \setminus \breve{h}_{\breve{M}}(n\omega_1 + m(1-n^{\eta})\omega_2) \tilde{\supseteq} \breve{U} \setminus \breve{h}_{\breve{M}}(\omega_1). \tag{59}$$

If $\breve{h}_{\breve{M}}(\omega_1) \subset \breve{h}_{\breve{M}}(\omega_2)$ then we may write

$$\breve{U} \setminus \breve{h}_{\breve{M}}(n\omega_1 + m(1-n^{\eta})\omega_2) \tilde{\supseteq} \breve{U} \setminus \breve{h}_{\breve{M}}(\omega_2). \tag{60}$$

From (24) *and* (25), we have

$$\breve{U} \setminus \breve{h}_{\breve{M}}(n\omega_1 + m(1-n^{\eta})\omega_2) \tilde{\supseteq} (\breve{U} \setminus \breve{h}_{\breve{M}}(\omega_1)) \tilde{\cap} (\breve{U} \setminus \breve{h}_{\breve{M}}(\omega_2)). \tag{61}$$

So, S^c is a m-convex soft set in the first sense. □

Theorem 4.13. *$\breve{M}$ is a m-concave soft set in the first sense on $\breve{L}$ if and only if for every $n \in [0,1]$ and $\breve{\delta} \in \breve{P}(\breve{U})$, $\breve{M}^{\breve{\delta}}$ is a m-concave set in the first sense on $\breve{L}$.*

Proof. Suppose $\breve{M}$ is a m-concave soft set in the first sense. If $\omega_1, \omega_2 \in \breve{L}$ and $\breve{\delta} \in \breve{P}(\breve{U})$,then $\breve{h}_{\breve{M}}(\omega_1) \tilde{\supseteq} \breve{\delta}$ and $\breve{h}_{\breve{M}}(\omega_2) \tilde{\supseteq} \breve{\delta}$. It follows from the m-concavity of $\breve{M}$ in the first sense that

$$\breve{h}_{\breve{M}}(n\omega_1 + m(1-n^{\eta})\omega_2) \tilde{\subseteq} \breve{h}_{\breve{M}}(\omega_1) \tilde{\cup} \breve{h}_{\breve{M}}(\omega_2) \tag{62}$$

and thus $\breve{M}^{\breve{\delta}}$ is a m-concave set in the first sense.
Conversely suppose that $\breve{M}^{\breve{\delta}}$ is a m-concave set in the first sense for every $\rho \in [0,1]$. For $\omega_1, \omega_2 \in \breve{L}$, $\breve{M}^{\breve{\delta}}$ is m-concave for $\breve{\delta} = \breve{h}_{\breve{M}}(\omega_1) \tilde{\cup} \breve{h}_{\breve{M}}(\omega_2)$. Since $\breve{h}_{\breve{M}}(\omega_1) \tilde{\subseteq} \breve{\delta}$ and $\breve{h}_{\breve{M}}(\omega_2) \tilde{\subseteq} \breve{\delta}$, we have $\omega_1 \in \breve{M}^{\breve{\delta}}$ and $\omega_2 \in \breve{M}^{\breve{\delta}}$, hence $n\omega_1 + m(1-n^{\eta})\omega_2 \in \breve{M}^{\breve{\delta}}$.Therefore, $\breve{h}_{\breve{M}}(n\omega_1 + m(1-n^{\eta})\omega_2) \tilde{\subseteq} \breve{\delta} = \breve{h}_{\breve{M}}(\omega_1) \tilde{\cup} \breve{h}_{\breve{M}}(\omega_2)$, which indicates $\breve{M}$ is a m-concave set in the first sense on $\breve{X}$. □

Theorem 4.14. *$\breve{M}$ is a m-concave soft set in the second sense on $\breve{L}$ if and only if for every $n \in [0,1]$ and $\breve{\delta} \in \breve{P}(\breve{U})$, $\breve{M}^{\breve{\delta}}$ is a m-concave set in the second sense on $\breve{L}$.*

Proof. Suppose that $\breve{M}$ is a m-concave soft set in the second sense. If $\omega_1, \omega_2 \in \breve{L}$ and $\breve{\delta} \in \breve{P}\left(\breve{U}\right)$,then $\breve{h}_{\breve{M}}(\omega_1) \tilde{\supseteq} \breve{\delta}$ and $\breve{h}_{\breve{M}}(\omega_2) \tilde{\supseteq} \breve{\delta}$. It follows from the m-concavity of $\breve{M}$ that

$$\breve{h}_{\breve{M}}\left(n\omega_1 + m(1-n)^{\eta}\,\omega_2\right) \tilde{\subseteq} \breve{h}_{\breve{M}}(\omega_1) \tilde{\cup} \breve{h}_{\breve{M}}(\omega_2) \tag{63}$$

and thus $\breve{M}^{\breve{\delta}}$ is a m-concave set in the second sense.
Conversely suppose that $\breve{M}^{\breve{\delta}}$ is a m-concave set in the second sense for every $n \in [0,1]$. For $\omega_1, \omega_2 \in \breve{L}$, $\breve{M}^{\breve{\delta}}$ is m-concave in the second sense for $\breve{\delta} = \breve{h}_{\breve{M}}(\omega_1) \tilde{\cup} \breve{h}_{\breve{M}}(\omega_2)$. Since $\breve{h}_{\breve{M}}(\omega_1) \tilde{\subseteq} \breve{\delta}$ and $\breve{h}_{\breve{M}}(\omega_2) \tilde{\subseteq} \breve{\delta}$, we have $\omega_1 \in M^{\breve{\delta}}$ and $\omega_2 \in M^{\breve{\delta}}$, hence $n\omega_1 + m(1-n)^{\eta}\,\omega_2 \in \breve{M}^{\breve{\delta}}$.Therefore, $\breve{h}_{\breve{M}}\left(n\omega_1 + m(1-n)^{\eta}\,\omega_2\right) \tilde{\subseteq} \breve{\delta} = \breve{h}_{\breve{M}}(\omega_1) \tilde{\cup} \breve{h}_{\breve{M}}(\omega_2)$, which indicates $\breve{M}$ is a m-concave set in the second sense on $\breve{X}$. □

5. Conclusion

In this work, *m*-convex and *m*-concave soft sets are developed along with their generalized set theoretic operations. Moreover, some classical approaches i.e. first and second senses, are utilized to get more generalized versions of *m*-convex and *m*-concave soft sets. Certain types of convexity like triangular convexity, graded convexity etc. can be introduced with the extension of this work for uncertain setting in future.

Competing interests

The authors declare that they have no competing interests.

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Author's contributions

All authors equally contributed to this work. All authors read and approved the final manuscript.

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