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INVESTIGATING THE PERFORMANCE OF PROGRESSIVE MEAN CHART BASED ON MEDIAN RUN LENGTH

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Abstract. The run length distribution of progressive mean (PM) control charts is highly skewed when the process is in-control or there is small shift in the process, so the interpretation based on average run length (ARL) may not be suitable. Furthermore, skewness varies at different shifts which cause difficulty in its interpretation on the basis of ARL. In the presence of skewness in run length, median run length (MRL) is the best approach for interpretation and accuracy in monitoring the process. The discussion of run length on the base of MRL is quite easy and readily understood. In this article, performance of PM control chart is evaluated in MRL and it is compared with ARL at the same shifts. This article indicates that MRL is more versatile average for understanding and explaining the distribution of run length because it is quick detector in case of small and moderate shift (highly and moderately skewed run length distribution) and gives good results in case of large shifts (symmetrical run length distribution). Along with standard deviation of run length (SDRL) is computed which is showing higher variability in process when shift is small but in case of moderate and large shifts smaller values of (SDRL). The performance of proposed control is compared with existing optimal EWMA control charts based on MRL and optimal CUSUM control charts based on MRL. The proposed control chart found to be more efficient than competitors.
Keywords: Average run length (ARL), Control charts, Progressive mean (PM), Median run length (MRL).

1. Introduction

All the manufacturing processes contain variation in their product. This variation can be further divided into two kinds called random causes of variation and assignable causes of variation. In the presence of random causes of variation process is known to be in-control (IC) but it is said to be out-of-control (OOC) when working under assignable causes of variation (cf. Nazir *et al*. [1], Abbas *et al*. [2] and Ali *et al.* [3]). Control charts are oriented for detection and removal of assignable causes of variation during the manufacturing processes. According to design structures of control charts; control charts are categorized into memory-less and memory-type. The memory-less charting model was originated by Shewhart in 1920s and it uses only recent information in the sample. The cumulative sum (CUSUM) originated by Page [4], exponentially weighted moving average (EWMA) designed by Roberts [5] and progressive mean (PM) suggested by Abbas *et al.* [6] are called memory-type charting mechanisms because these use past and current information during execution. In last few years PM control charts have gained lot of attention for detecting quickly small and moderate shifts in manufacturing process. Abbas *et al.* [6] developed PM control charts for detecting small shifts in the process mean on the basis

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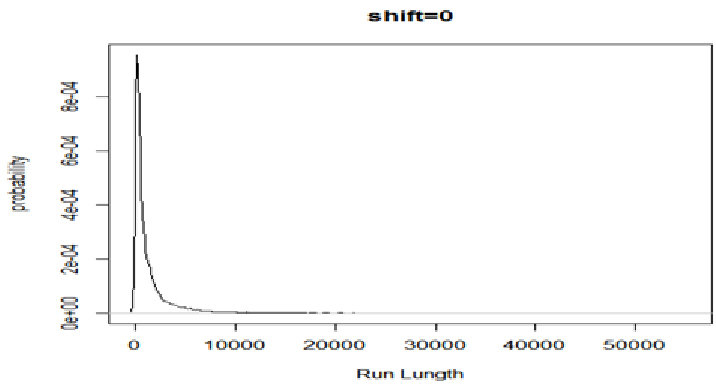
of ARL. Abbasi *et al.* [7] proposed non-parameter PM charts for on-line monitoring of the process median and comparisons are made on the base of ARL. Zafar *et al.* [8] developed progressive variance charts for examining the shifts in process variance on the behalf of ARL. Readers are referred to Abbas *et al.* [9]-[10], Riaz *et al.* [11] and Abbas *et al.* [12] for the recent developments using PM structures. It can be observed that in the stated PM chart all the comparisons are made with the existing counterparts using ARL of run length (RL) distribution. Gan [13] declared that description of ARL for highly skewed RL distribution is not similar with symmetric RL distribution. So, in case of skewed RL distribution the ARL does not present the true picture of manufacturing process because it confuses the practitioners and engineers. For the stated reasons, Jones *et al.* [14], Jensen *et al.* [15], Bischak and Trietsch[16] have criticised the ARL as measure of performance in control charts during process monitoring. To overcome the above submitted problems with ARL measure, Lai [17] and Chakraborti [18] suggested to use median run length (MRL) instead of ARL for measuring, comparing and interpretation of different properties of charts. Gan [13] proposed an optimal EWMA control charts and Gan [19] proposed an optimal CUSUM control charts based on MRL. The MRL is the median number of observations of RL distribution before a chart gives OOC point or the $50^t h$ percentage signal of the RL is called MRL. Taking inspiration from the above, in this article PM chart is proposed on the basis of MRL because in literature there is a huge vacuum of it. It may help quality practitioners and quality engineers for further developments. It is observed in the Figure 1, that the RL distribution of PM control charts is highly skewed when the process is IC (shift=0) and when shift $s = 0.25, 0.50$. It is approximately symmetrical in case of small shifts 1.00 but when the shift becomes larger (more than 1.00) there are dramatically changes in the RL distribution of PM charts. When shifts are 2.00 and 3.00 the RL distribution of PM control chart is multi-modal. In case of shifts 4.00 and 5.00 the RL distribution of PM control chart has tri-modal and bi-modal respectively (cf. Figure 1). Furthermore, from the eight graphs (cf. Figure 1) it can be seen that skewness of RL of distribution in PM control chart changes as the shifts changes. It is obvious that ARL interpretation of symmetrical RL distribution is not same for the ARL of skewed RL distribution. The use of ARL is only suitable when the RL distribution has same skewness otherwise its interpretation would be misleading and difficult to understand. The MRL is free from such difficulty of interpretation when the distribution has variety in skewness. Gan [19] noted that use of MRL makes readily and clearly understanding of control chart to quality control engineers and practitioners. In short, MRL of 370 can be taken such as 50 percent of all the RLs are less than 370" or "half of RLs are below 370". Similarly, for OOC MRL with value 28 is mean that below 28 RLs are half. The appropriate average under the different skewness in RL of distribution when there are shifts in mean is MRL because it is more meaningful, reliable and easy to interpret. In this article, as the graphs of Figure 1 shows not same skewness in the RL distribution of PM control charts, so the MRL is proposed to measure the performance of control charts instead of ARL for the meaningful interpretation.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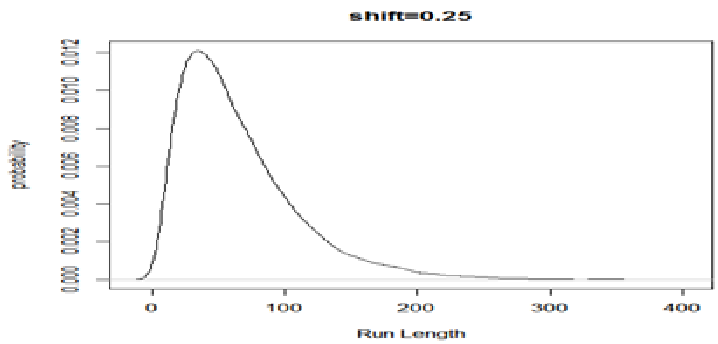
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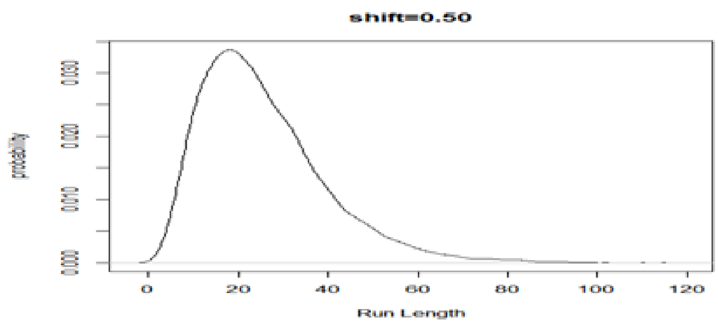
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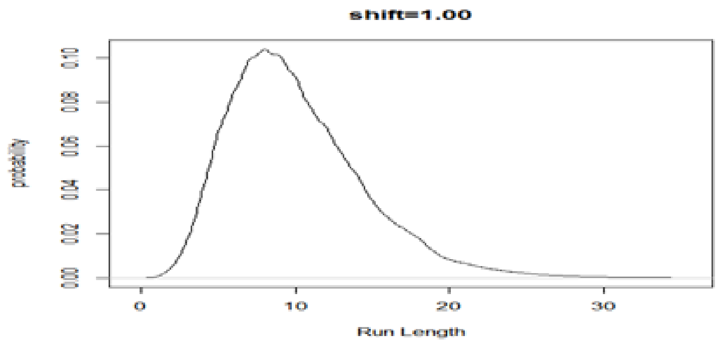
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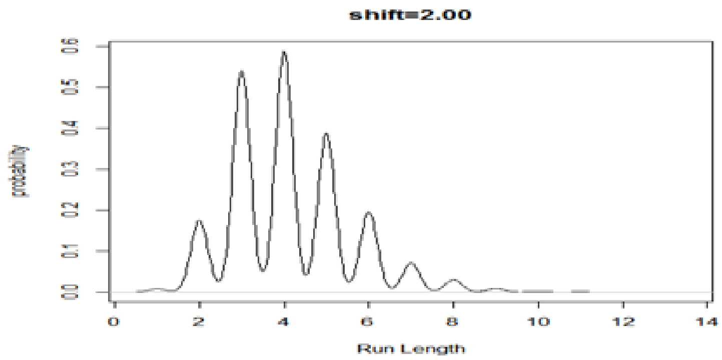
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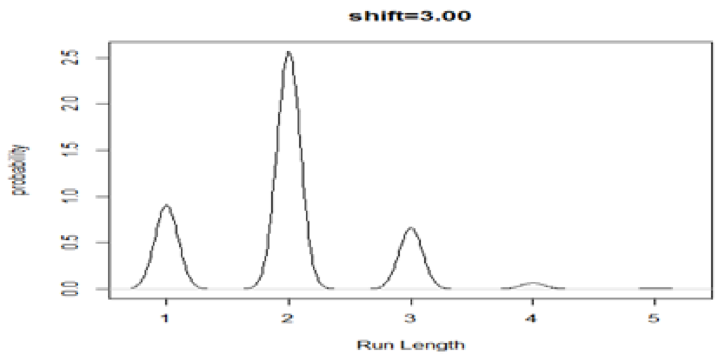
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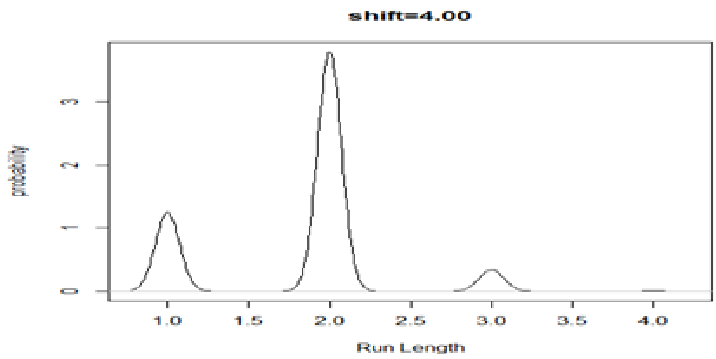

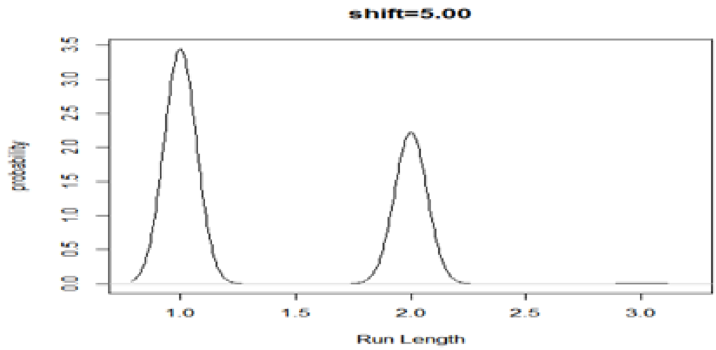

FIGURE 1. Graphs of probability distribution of Run length at different shifts(shifts= $0, 0.25, 0.50, 1.00, 2.00, 3.00, 4.00 and 5.00$) for PM control charts with $C = 1.583$.

2. AN OVERVIEW OF PM CONTROL CHART

Suppose Y is the quality characteristic of interest which is being monitored in the manufacturing process. In this article, an individual observation is taken from the normal distribution. Let $Y_i, i = 1, 2, 3,$ is the set of independently and identically distributed observations from the process which is being monitored, then the PM is defined as the cumulative average of observations over time. The PM statistic can be written mathematically as

$$PM_j = \frac{\sum_{j=1}^{i} Y_j}{i} \tag{1}$$

The PM in 1 shows that it is cumulative average because it includes next observation and does not exclude previous observations like moving average. The mean and variance of PM. Estimators are $E(PM) = \mu_0$ and

$Var(PM) = \frac{\sigma^2}{i}$, where μ_0 and σ_0^2 are in-control mean and variance of the process. The three sigma control limits of PM control chart are presented as

$$LCL_i = \mu_0 - 3\frac{\sigma_0}{\sqrt{i}}, \qquad CL = \mu_0, \qquad UCL_i = \mu_0 + 3\frac{\sigma_0}{\sqrt{i}} \tag{2}$$

The above 2, shows that limits are time varying and consist of equal weights to current and previous information. The control limits become larger for larger value of i ($i > 1000$) which cause small chances of getting out-of-control point. The problem was solved by Abbas *et al.* [6] after imposing penalty of $f(i) = i^0.20$ and get the executed control limits as below

$$LCL_i = \mu_0 - 3\frac{\sigma_0}{\sqrt{i}}(\frac{C}{f(i)}), \qquad CL = \mu_0, \qquad UCL_i = \mu_0 + 3\frac{\sigma_0}{\sqrt{i}}(\frac{C}{f(i)}) \tag{3}$$

Where C is constant which controls the RLs of distribution. For monitoring the performance of suggested control chart in this article MRL is used as performance measure. To obtain the run length (RL) properties of the control structure Monte Carlo simulations has been applied. To represent the state of the process δ is used; when $\delta = 0$ (the process is said to be in-control) and $\delta \neq 0$ (the process is out-of-control). For evaluating the performance of different properties of RL, in this article $10,000$ simulation runs are applied. The evaluation of median run length of distribution is computed as

$$Pr(RL < MRL) < 0.50;$$

For some predefined values of $MRL = 200, 370, 500$ the constant values of C are computed. The values of MRL_0 and MRL_1 are computed, these values are presented in Table 1. To discuss some other properties of RL distribution $ARLs$ are also computed at pre-specified values of MRL.

Table 1. MRL values of PM control chart under different shifts

δ	C =1.293 $MRL = 200$		$C = 1.485$ $MRL = 370$		$C = 1.583$ $MRL = 500$	
	MRL	ARL	MRL	ARL	MRL	ARL
0.00	200	552.03	369	1067.29	498	1398.50
0.10	99	150.29	142.5	198.75	163	223.00
0.20	51	65.6583	65	81.89	74	90.93
0.30	31	38.38	40	47.25	44	52.24
0.40	22	26.19	28	32.097	31	34.83
0.50	17	19.26	21	23.46	23	25.70
0.60	13	15.26	17	18.48	18	20.03
0.70	11	12.54	13	14.82	15	16.41
0.80	9	10.52	11	12.46	13	13.61
0.90	8	8.92	10	10.63	11	11.728
1.00	7	7.76	9	9.31	9	10.11
1.10	6	6.89	8	8.14	8	8.94
1.20	6	6.16	7	7.25	7	7.92
1.30	5	5.55	6	6.54	7	7.10
1.40	5	5.02	6	5.97	6	6.52
1.50	4	4.64	5	5.46	6	5.92
1.60	4	4.24	5	5.00	5	5.43
1.70	4	3.92	4	4.65	5	5.032
1.80	3	3.65	4	4.35	5	4.7098
1.90	3	3.46	4	4.04	4	4.395
2.00	3	3.22	4	3.77	4	4.0757
3.00	2	2.03	2	2.34	2	2.4972
4.00	1	1.46	2	1.72	2	1.8491
5.00	1	1.13	1	1.29	1	1.3943

Following Palm [20], Shmueli and Cohen [21], Antzoulakos and Rakitzis [22] and Abbas *et al.* [23] SDRL and some percentile points are also evaluated in this study to monitor the true picture of RLs. The reported standard deviation of run length (SDRL) and percentile points $(P_{25} and P_{75})$ of RLs in Table 2 are evaluated on the same constant values presented in Table 1. From the Tables $1-2$, following conclusions are made after evaluating PM control chart on the base of MRL:

- The performance of proposed PM control chart is clearly efficient in detecting small, moderate and large shifts in the process (cf. Table 1).
- For pre-fixed value of MRL $(MRL = 200, 370 and 500)$, ARL_0 is greater than MRL_0 (cf. Table 1).
- For the small shifts in the mean of process $(\delta = 0.10 to 0.70)$ MRL gives out-of-control signal very quickly than ARL (cf. Table 1).
- When the shifts in the process is of moderate size $(\delta = 0.80 to 1.30)$, MRL gives out-of-control signal comparatively faster as compare to ARL (cf. Table 1).
- For large shifts in the process mean $(\delta = 1.40 to 5.00)$ both the averages perform almost identically (cf. Table 1).

Table 2. SDRL and Percentile Points of PM control chart are different shifts

δ	C =1.293 $MRL = 200$			$C = 1.485$ $MRL = 370$			$C = 1.583$ $MRL = 500$		
	SDRL	P_{25}	P_{75}	SDRL	P_{25}	P_{75}	SDRL	P_{25}	P_{75}
0.00	1048.73	67	573	2233.92	121	1073	2714.89	166	1458.75
0.10	155.75	44	202	187.40	66	268	201.73	81	300.75
0.20	53.39	27	88	62.81	37	109	67.76	42	121
0.30	27.79	18	51	31.57	24	62	33.66	28	69
0.40	17.17	14	35	19.57	18	42	20.26	20	45
0.50	11.72	11	25	13.16	14	30	13.97	16	33
0.60	8.79	9	20	9.78	11	24	10.12	13	26
0.70	6.91	7	16	7.60	9	19	7.96	11	21
0.80	5.45	6	13	5.98	8	16	6.36	9	17
0.90	4.52	6	11	4.88	7	13	5.20	8	15
1.00	3.75	5	10	4.20	6	12	4.36	7	13
1.10	3.21	5	9	3.56	6	10	3.72	6	11
1.20	2.81	4	8	3.03	5	9	3.20	6	10
1.30	2.43	4	7	2.67	5	8	2.75	5	9
1.40	2.15	3	6	6	5.96	7	2.50	5	8
1.50	1.91	3	6	2.11	4	7	2.22	4	7
1.60	1.72	3	5	1.89	4	6	1.97	4	7
1.70	1.56	3	5	1.69	3	6	1.798	4	6
1.80	1.41	3	4	1.55	3	5	1.625	4	6
1.90	1.33	3	4	1.42	3	5	1.495	3	5
2.00	1.18	2	4	1.29	3	4	1.3717	3	5
3.00	0.65	2	2	0.704	2	3	0.7287	2	3
4.00	0.56	1	2	0.53	1	2	0.5131	2	2
5.00	0.33	1	1	0.456	1	2	0.4914	1	2

- The MRL_1 of proposed control chart decreases rapidly as the δ in the mean of process increases (cf. Table 1).
- The run length distribution of proposed control chart is positively skewed (cf. Table 1).
- The standard deviation of run length (SDRL) is very high at $\delta = 0$ and it decreases as the δ becomes larger (SDRL is inversely proportion to shifts) during process (cf. Table 2).

- As the MRL and ARL become close to each other SDRL becomes smaller and vice versa (cf. Table $1-2$).
- At $\delta = 0.10$ and $C = 1.583$ the values of $MRL = 163$ and $ARL = 223$, the values of $P_{25} = 81$ and $P_{75} = 300.75$ with $SDRL = 201.73$ which shows skewness in the RL of distribution (cf. Table 1 and Table 2).

3. Comparisons

In every process there is variation which needs to detect and remove from the process. For large shifts Shewhart control charts are used and to detect small and moderate kind of shifts in the process EWMA and CUSUM control charts are applied. The performance of control charts is commonly monitored on the basis of ARL but Figure 1 shows that the RLs of distribution have different nature on variety of shifts. In this situation when the shape of RLs distribution is not symmetrical ARL gives mis-leading results and its interpretation become difficult. In this proposed study, the performance of PM control chart is compared with optimal EWMA and optimal CUSUM control charts on the basis of MRL instead of commonly used ARL. In this proposed study, MRL_0 is taken at $200, 370$ and 500 to give valid comparison with each existing counterparts control charts. The comparisons are presented in the following sections of proposed PM control chart with its existing competitors.

3.1. **Proposed versus Optimal EWMA.** Gan [13] proposed optimal EWMA control charts and evaluated its performance on the basis of MRL instead of ARL. The evaluated MRL values of optimal EWMA control charts are presented in Table 3. The Table 3 shows that the optimal EWMA control charts give similar $MRL_1 = 10$ at $\delta = 1.00$ at which rapidly detection has importance. Comparing it with proposed PM control chart at $\delta = 1.00$ the value of $MRL_1 = 9$, which is quick signal by the proposed control chart (cf. Table 1 and Table 3). From the Table 3, in optimal EWMA control charts at $MRL_0 = 500$ with parameters $\delta = 0.130$ and $h = 0.792$ it is observed that all the values of MRL_1 are the smallest at variety of shifts ($\delta = 0.10 to 2.00$) than any other combination of parameters. Now, comparing with the proposed PM control chart presented in Table 1, it is clear that proposed chart performs far better than optimal EWMA at each kind of shift small, moderate or large (cf. Table 1 and Table 3).

Table 3. Values of *MRL* in Optimal EWMA control charts

δ	λ=0.130	λ=0.145	λ=0.160	λ=0.175	λ=0.190	λ=0.205	λ=0.220
	h=0.792	h=0.846	h=0.897	h=0.947	h=0.996	h=1.043	h=1.089
0.00	500	500	500	500	500	500	500
0.10	327	335	344	351	358	364	370
0.20	154	162	170	178	186	193	201
0.30	78	183	88	93	98	103	107
0.40	46	148	51	53	56	59	62
0.50	30	31	33	34	36	37	39
0.60	22	22	23	24	24	25	26
0.70	17	17	17	18	18	19	19
0.80	13	14	14	14	14	14	15
0.90	11	11	11	11	11	12	12
1.00	10	10	10	10	10	10	10
1.10	9	8	8	8	8	8	8
1.20	8	7	7	7	7	7	7
1.30	7	7	7	7	6	6	6
1.40	6	6	6	6	6	6	6
1.50	6	6	5	5	5	5	5
1.60	5	5	5	5	5	5	5
1.70	5	5	5	5	5	4	4
1.80	5	4	4	4	4	4	4
1.90	4	4	4	4	4	4	4
2.00	4	4	4	4	4	4	4
3.00	3	3	3	2	2	2	2
4.00	2	2	2	2	2	2	2
5.00	2	2	2	2	2	2	1

The optimal EWMA control charts with the best combination of parameters ($\delta = 0.130$ and $h = 0.792$) give $MLR_1 = 327$,30 and 17, while the proposed PM control chart give $MRL_1 = 163, 23$, when the process is working under same shifts respectively ($\delta = 0.10, 0.50 and 0.70$). From the discussion of results provided in Table 1 and Table 3 it is obvious that the proposed PM control chart is superior in detecting shifts on the basis of MRL than optimal EWMA control charts.

3.2. **Proposed versus Optimal CUSUM.** Gan [19] suggested optimal CUSUM control charts on the basis of MRL. The profile evaluation of $MRL_0 = 500$ is presented in Table 4 at different combinations of parameters under small, moderate and large shifts in the process mean. From Table 4, it can be noted that at smaller values of “k” the optimal CUSUM control charts detect small shifts very quickly but when “k” is large the performance of optimal CUSUM control charts becomes efficient for large shifts in the process. In the Table 4, optimal CUSUM control charts have optimal value of $MRL_1 = 9$ between the various values of k $[0.40-0.90]$. To compare the performance of proposed PM control chart and optimal CUSUM control charts, it can be observed that with parameters $\lambda = 1.20$, $h = 2.066$ at $\delta = 0.20$, optimal CUSUM control charts give $MRL_1 = 206$ but proposed chart gives $MRL_1 = 74$ on the same shift (cf. Table 1 and Table 4). The proposed PM control chart also gives $MRL_1 = 9$ at $\delta = 1.00$ which is same with optimal range of k $[0.40-0.50]$ with optimal CUSUM (cf. Table 1 and Table 4).

Table 4. MRL Values of $CUSUM$ control charts

δ	k	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00	1.10	1.20	1.30	2.99
	h	13.272	9.159	7.009	5.669	4.745	4.065	3.541	3.124	2.784	2.503	2.267	2.066	1.891	0.00
0.00		500	500	500	500	500	500	500	500	500	500	500	500	500	500
0.10		158	174	195	215	234	250	264	277	289	300	310	319	327	362
0.20		80	81	91	104	118	132	146	159	171	183	195	206	216	265
0.30		52	48	51	57	65	74	84	94	104	114	124	134	144	195
0.40		39	33	33	35	40	45	51	58	65	73	81	89	97	145
0.50		31	25	24	25	26	20	33	38	43	48	54	60	67	109
0.60		25	20	19	18	19	21	23	26	29	33	37	42	46	83
0.70		22	17	15	15	15	16	17	18	21	23	26	29	33	63
0.80		19	15	13	12	12	12	13	14	15	17	19	21	24	49
0.90		17	13	11	10	10	10	10	11	12	13	14	16	18	38
1.00		15	12	10	9	9	9	9	9	9	10	11	12	14	36
1.10		14	10	9	8	8	7	7	8	8	8	9	10	11	24
1.20		12	9	8	7	7	7	6	6	7	7	7	8	9	19
1.30		11	9	7	7	6	6	6	6	6	6	6	7	7	15
1.40		11	8	7	6	6	5	5	5	5	5	5	6	6	13
1.50		10	7	6	6	5	5	5	5	5	5	5	5	5	10
1.60		9	7	6	5	5	4	4	4	4	4	4	4	4	9
1.70		9	7	5	5	4	4	4	4	4	4	4	4	4	7
1.80		8	6	5	5	4	4	4	4	3	3	3	3	4	6
1.90		8	6	5	4	4	4	3	3	3	3	3	3	3	5
2.00		7	6	5	4	4	3	3	3	3	3	3	3	3	4
3.00		5	4	3	3	2	2	2	2	2	2	2	2	2	1
4.00		4	3	2	2	2	2	2	1	1	1	1	1	1	1
5.00		3	2	2	2	2	1	1	1	1	1	1	1	1	1

Finally, from Table 1 and Table 4, it is obvious that proposed control chart is performing efficiently for detection of small, moderate and large shifts in the process mean.

3.3. **Real Life Example.** To illustrate the PM chart based on MRL the real life data is taken from Montgomery [24] related to the velocity of light in air using a modification of a method proposed by the French physicist Foucauld.

Table 5. Individual Measurements with Corresponding Z, PM and Control limits

Measurements		Z	PM	LCL	UCL
I	Velocity(X)				
1	850	-0.36551	-3.66E-01	-4.249	5.249
2	1000	1.321442	4.78E-01	-2.42335	3.423352
3	740	-1.6026	-2.16E-01	-1.70099	2.700986
4	980	1.096516	1.12E-01	-1.29953	2.299535
5	900	0.196811	1.29E-01	-1.0393	2.0393
6	930	0.5342	1.97E-01	-0.85487	1.854866
7	1070	2.108684	4.70E-01	-0.71628	1.71628
8	650	-2.61477	8.43E-02	-0.60774	1.607743
9	930	0.5342	1.34E-01	-0.52008	1.520076
10	760	-1.37767	-1.69E-02	-0.44755	1.44755
11	850	-0.36551	-4.86E-02	-0.38639	1.386395
12	810	-0.81536	-1.12E-01	-0.33402	1.334018
13	950	0.759126	-4.54E-02	-0.28857	1.288573
14	1000	1.321442	5.22E-02	-0.24871	1.248708
15	980	1.096516	1.22E-01	-0.21341	1.213409
16	1000	1.321442	1.97E-01	-0.1819	1.181896
17	980	1.096516	2.50E-01	-0.15356	1.153564
18	960	0.87159	2.84E-01	-0.12793	1.12793
19	880	-0.02812	2.68E-01	-0.10461	1.104609
20	960	0.87159	2.98E-01	-0.08329	1.083286
21	960	0.87159	3.25E-01	-0.0637	1.063701
22	830	-0.59043	2.84E-01	-0.04564	1.04564
23	940	0.646663	2.99E-01	-0.02892	1.028923
24	790	-1.04028	2.44E-01	-0.0134	1.013398
25	960	0.87159	2.69E-01	0.001065	0.998935
26	810	-0.81536	2.27E-01	0.014576	0.985424
27	940	0.646663	2.43E-01	0.027233	0.972767
28	880	-0.02812	2.33E-01	0.039116	0.960884
29	880	-0.02812	2.24E-01	0.050299	0.949701
30	880	-0.02812	2.16E-01	0.060846	0.939155
31	800	-0.92782	1.79E-01	0.070811	0.929189
32	830	-0.59043	1.55E-01	0.080244	0.919756
33	850	-0.36551	1.39E-01	0.089189	0.910811
34	800	-0.92782	1.08E-01	0.097684	0.902316
35	880	-0.02812	1.04E-01	0.105766	0.894234
36	790	-1.04028	7.19E-02	0.113464	0.886536
37	900	0.196811	7.52E-02	0.120806	0.879194
38	760	-1.37767	3.70E-02	0.127819	0.872181
39	840	-0.47797	2.38E-02	0.134526	0.865474
40	800	-0.92782	7.06E-18	0.140946	0.859054

Forty measurements are reported in Table 5 to illustrate the PM control chart based on MRL. From these measurements PM control chart based on MRL is investigated at $MRL_0 = 500, C = 1.583$ when a $shift = 0.50$ is introduced in the process. First of all measurements are standardized for computing PM statistic and the both varying control limits. The further computation is presented in the Table 5.

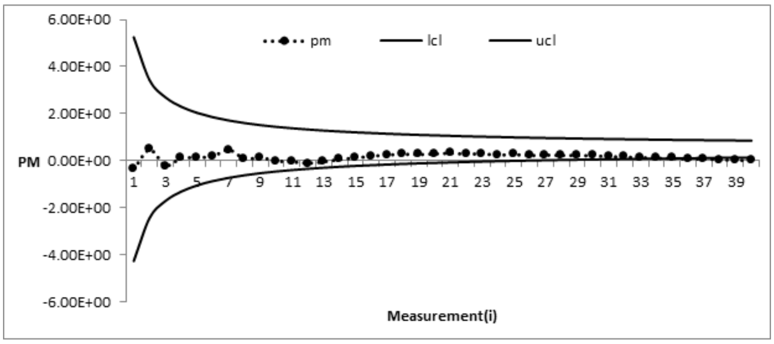

FIGURE 2. Graphical presentation of Proposed PM control chart based on MRL

The Figure 2 is designed from the entries in Table 5 in which y-axis is showing computed values of PM statistic and x-axis is presenting sample numbers. It is clear from Figure 2 that the process is OOC detected by PM chart at sample numbers 34-40 (7 points OOC). To investigate the underlying process for detection of assignable causes, further actions can be applied.

4. CONCLUSION

To detect and avoid the unnatural variation during a process, control charts plays significant rule. According to their performance these are divided into two categories called memory less and memory-type control charts. For detecting small shifts more quickly memory-type control charts like EWMA, CUSUM and PM control charts are used. This paper presents the PM control chart to monitor shift in mean of process on the basis of MRL criterion because when there is not symmetry in the run length of distribution ARL gives misleading and false interpretation of RLs of distribution. The MRL is more clearly understandable by users even it is from extremely skewed RL distribution. The efficiency of proposed control chart is compared with existing two memory-type; optimal EWMA and optimal CUSUM control charts on the basis of MRL. The proposed PM control chart based on MRL is superior in detecting small, moderate and large shifts in the process mean as compare to optimal EWMA control charts based on MRL and optimal CUSUM control charts based on optimal control charts.

COMPETING INTERESTS

The authors declare that they have no competing interests.

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AUTHOR'S CONTRIBUTIONS

All authors equally contributed to this work. All authors read and approved the final manuscript.

REFERENCES

[1] Nazir, H. Z., Hussain, T., Akhtar, N., Abid, M. & Riaz, M. (2019). Robust adaptive exponentially weighted moving average control charts with applications of manufacturing processes. *The International Journal of Advanced Manufacturing Technology*, **105**(1-4), 733-748.

[2] Abbas, Z., Nazir, H. Z., Abid, M., Akhtar, N. & Riaz, M. (2020). Enhanced nonparametric control charts under simple and ranked set sampling schemes. *Transactions of the Institute of Measurement and Control*, **42**(14), 2744-2759.

[3] Ali, S., Abbas, Z., Nazir, H.Z., Riaz, M., Zhang, X., & Li, Y. (2020). On Designing Non-Parametric EWMA Sign Chart under Ranked Set Sampling Scheme with Application to Industrial Process. *Mathematics*, **8**(9).

[4] Page, E.S. (1954). Continuous inspection schemes. *Biometrika*, **41**(1/2), 100-115.

[5] Roberts, S. (1959). Control chart tests based on geometric moving averages. *Technometrics*, **1**(3), 239-250.

[6] Abbas, N., Zafar, R. F., Riaz, M. & Hussain, Z. (2013). Progressive mean control chart for monitoring process location parameter. *Quality and Reliability Engineering International*, **29**(3), 357-367.

[7] Abbasi, S. A., Miller, A. & Riaz, M. (2019). Nonparametric progressive mean control chart for monitoring process target. *Quality and Reliability Engineering International*, **29**(7), 1069-1080.

[8] Zafar, R. F., Abbas, N., Riaz, M., & Hussain, Z. (2014). Progressive variance control charts for monitoring process dispersion. *Communications in Statistics-Theory and Methods*, **43**(23), 4893-4907.

[9] Abbas, Z., Nazir, H. Z., Akhtar, N., Riaz, M., & Abid, M. (2019). An enhanced approach for the progressive mean control charts. *Quality and Reliability Engineering International*, **35**(4), 1046-1060.

[10] Abbas, Z., Nazir, H. Z., Akhtar, N., Riaz, M., & Abid, M. (2020). On developing an exponentially weighted moving average chart under progressive setup: An efficient approach to manufacturing processes. *Quality and Reliability Engineering International*, **36**(7), 2569-2591.

[11] Riaz, M., Abbas, Z., Nazir, H. Z., Akhtar, N., & Abid, M. (2020). On Designing a Progressive EWMA Structure for an Efficient Monitoring of Silicate Enactment in Hard Bake Processes. *Arabian Journal for Science and Engineering*, https://doi.org/10.1007/s13369-020-04948-y.

[12] Abbas, Z., Nazir, H. Z., Abid, M., Akhtar, N. & Riaz, M. (2020). Nonparametric progressive sign chart for monitoring process location based on individual data. *Quality Technology and Quantitative Management*, https://doi.org/10.1080/16843703.2020.1827726.

[13] Gan, F. (1993). An optimal design of EWMA control charts based on median run length. *Journal of Statistical Computation and Simulation*, **45**(3-4), 169-184.

[14] Jones, L. A., Champ, C. W. & Rigdon, S. E. (2004). The run length distribution of the CUSUM with estimated parameters. *Journal of Quality Technology*, **36**(1), 95-108.

[15] Jensen, W. A., Jones-Farmer, L. A., Champ, C. W., & Woodall, W. H. (2006). Effects of parameter estimation on control chart properties: a literature review. *Journal of Quality Technology*, **38**(4), 349-364.

[16] Bischak, D. P. & Trietsch, D. (2007). The rate of false signals in $\bar{U}$ control charts with estimated limits. *Journal of Quality Technology*, **39**(1), 54-65.

[17] Lai, T. L. (1995). Sequential changepoint detection in quality control and dynamical systems. *Journal of the Royal Statistical Society. Series B (Methodological)*, 613-658.

[18] Chakraborti, S. (2007). Run length distribution and percentiles: the Shewhart chart with unknown parameters. *Quality Engineering*, **19**(2), 119-127.

[19] Gan, F. (1994). An optimal design of cumulative sum control chart based on median run length. *Communications in Statistics-Simulation and Computation*, **23**(2), 485-503.

[20] Palm, A. C. (1990). Tables of run length percentiles for determining the sensitivity of Shewhart control charts for averages with supplementary runs rules. *Journal of Quality Technology*, **22**(4), 289-298.

[21] Shmueli, G. & Cohen, A. (2003). Run-length distribution for control charts with runs and scans rules. *Communications in Statistics-Theory and Methods*, **32**(2), 475-495.

[22] Antzoulakos, D. L., & Rakitzis, A. C. (2008). The modified r out of m control chart. *Communications in Statistics-Simulation and Computation*, **37**(2), 396-408.

[23] Abbas, N., M. Riaz, & Does, R. J. (2011). Enhancing the performance of EWMA charts. *Quality and Reliability Engineering International*, **27**(6), 821-33.

[24] Montgomery, D. C. (2012). *Introduction to statistical quality control* (7th edition). John Wiley and Sons, New York.